



Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

--	--	--	--	--	--	--	--

In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

Number of additional
answer booklets used
(if applicable):

--

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	13	13	100	99	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (99 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

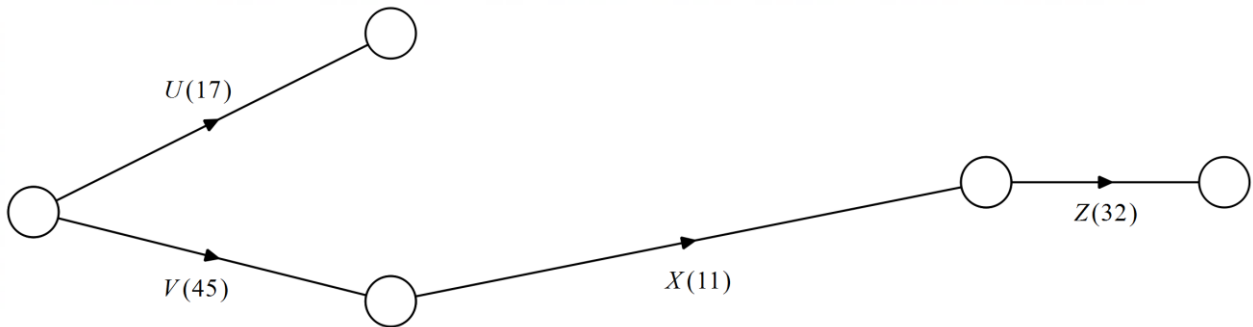
Question 9

(5 marks)

The table below shows the duration and immediate predecessors for all the activities required to prepare a meal.

Activity	<i>U</i>	<i>V</i>	<i>W</i>	<i>X</i>	<i>Y</i>	<i>Z</i>
Duration (minutes)	17	45	21	11	13	32
Immediate predecessors	–	–	<i>U</i>	<i>V</i>	<i>V, W</i>	<i>X, Y</i>

- (a) Complete the project network below to show all the above information. (2 marks)



- (b) If the meal is to be served at 7.15 pm, determine the latest time the preparations must begin so that the meal is served on time and the path of activities that cannot be delayed. (3 marks)

Question 10**(10 marks)**

The body length and eye diameter of eight tropical fish of the same species are shown in the table below.

Body length, L cm	6.3	11.9	3.8	8.2	9.1	10.5	5.2	7.5
Eye diameter, D mm	9.3	8.6	9.9	9.1	9.0	8.7	9.8	9.4

- (a) Use your calculator to graph the above data and hence describe the direction and strength of the linear association between the variables. (2 mark)
- (b) Determine the coefficient of determination for the linear association and interpret its value in context. (2 marks)
- (c) Determine the equation of the least-squares line to predict D from L and interpret the slope of the line in context. (3 marks)
- (d) Predict the eye diameter of another tropical fish of the same species that has a body length of 5.6 cm and give two reasons that support the validity of this prediction. (3 marks)

Question 11

(7 marks)

A builder bought a scissor lift for \$38 000 and for accounting purposes will depreciate its value by 15% annually.

(a) Show that the value of the lift after one year will be \$32 300. (1 mark)

(b) Write a recursive rule to calculate the value of the lift, T_n in dollars, after n years. (2 marks)

(c) Calculate the value of the lift after 4 years. (1 mark)

(d) Deduce the n^{th} term rule for the value of the lift after n years. (1 mark)

(e) Determine the least number of years that must pass for the lift to depreciate by more than 80% of its original value. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 12**(7 marks)**

A reducing balance loan is defined by the recurrence relation $T_{n+1} = a \times T_n - b$, $T_1 = c$, where T_n is the balance of the loan in dollars at the start of month n . The relation was used to create the following spreadsheet.

Month n	Balance of loan at start of month n	Monthly interest	Monthly repayment	Loan balance carried forward
1	16 500.00	140.25	425.00	16 215.25
2	16 215.25	137.83	425.00	15 928.08
3	15 928.08	135.39	425.00	X
4		Y		Z

(a) Determine the annual percentage interest rate that applies to the loan. (2 marks)

(b) State the value of each of the constants a , b and c in the recurrence relation. (2 marks)

(c) Determine the value of X , the value of Y and the value of Z shown in the spreadsheet. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 13

(9 marks)

A researcher observed a large number of cats, noting for each one its gender and the paw that it used to react to a stimulus placed in front of it. The numbers in each category are shown in the table below.

Cats	Left-pawed	Right-pawed
Male	27	31
Female	28	55

The researcher is interested in whether there is an association between the variables.

- (a) Name one of the variables in the study and classify it as numerical or categorical. (2 marks)

- (b) Determine the percentage of left-pawed cats that were female. (2 marks)

- (c) Use the above data to complete the following table so that it shows the appropriate column or row percentages rounded to the nearest whole number. (3 marks)

Cats (%)	Left-pawed	Right-pawed
Male		
Female		

- (d) Explain whether the percentaged table suggests the presence of an association between the variables. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 14

(8 marks)

The table below shows the number of new clients attracted by a business during the first three weeks of an advertising campaign, together with some derived values to assist in the analysis of the time series data.

Week	Day (<i>n</i>)		New Clients	Weekly mean	Percentage of weekly mean
1	Mon	1	35	40.6	A
	Tue	2	40		98.5
	Wed	3	44		108.4
	Thu	4	43		105.9
	Fri	5	41		101.0
2	Mon	6	42	B	89.7
	Tue	7	44		94.0
	Wed	8	52		111.1
	Thu	9	49		104.7
	Fri	10	47		100.4
3	Mon	11	49	55.2	88.8
	Tue	12	55		99.6
	Wed	13	59		106.9
	Thu	14	56		101.4
	Fri	15	C		103.3

(a) Calculate the value of **A**, the value of **B** and the value of **C** in the table. (3 marks)

(b) Determine the seasonal index for Thursdays. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

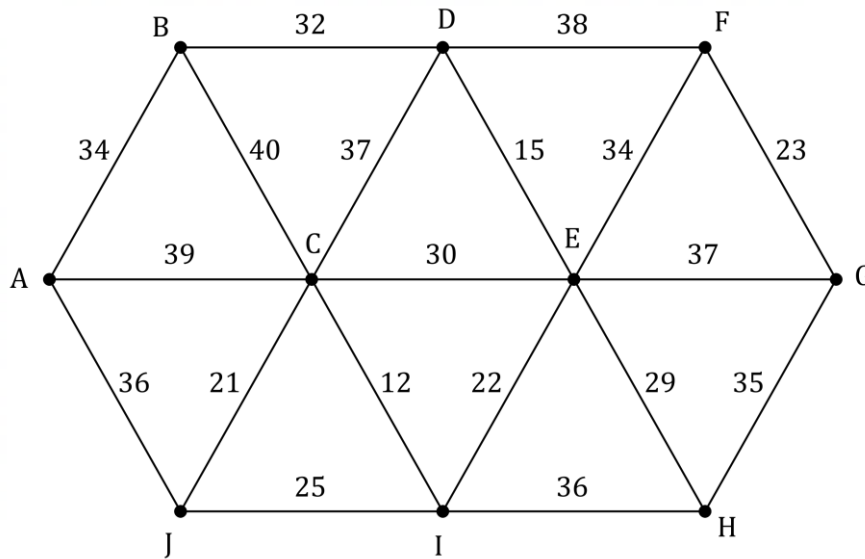
- (c) The least-squares line to predict the deseasonalised number of new clients c from the day is $c = 1.31n + 37.1$. Determine the best estimate for the number of new clients expected on Wednesday of Week 4. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 15

(8 marks)

Ten mains-powered smoke alarms must be installed in a building. The edge weights in the graph below represent the length of cable, in metres, required between adjacent alarms.

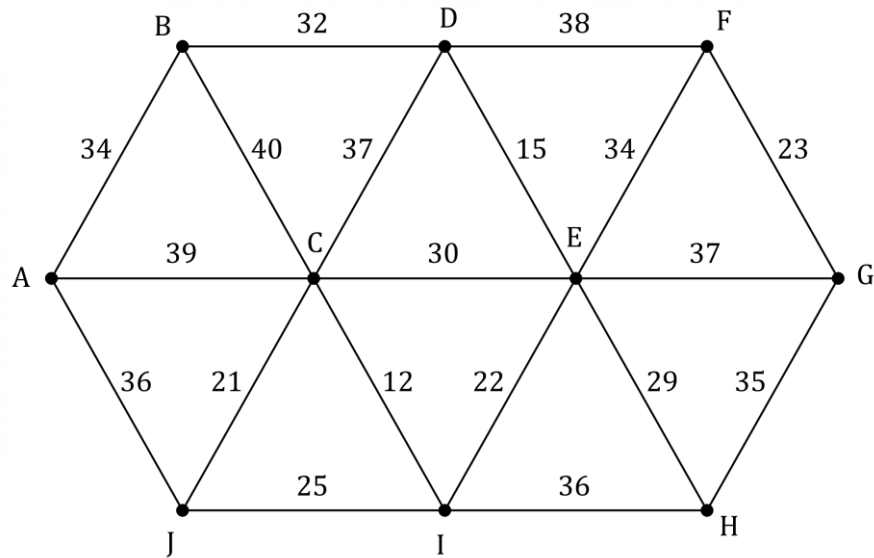


- (a) Clearly indicate the minimum spanning tree on the graph above. (3 marks)

- (b) Determine the cost of installing the cabling between the alarms using the minimum spanning tree, given that each metre of cabling will cost \$18.50. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

- (c) Explain how your answer to part (b) will change if smoke alarm *K* is added to the system with cable lengths of 20, 21 and 22 metres to alarms *F*, *G* and *H* respectively. (A copy of the graph from the previous page is shown below if you wish to use it.) (3 marks)



DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 16

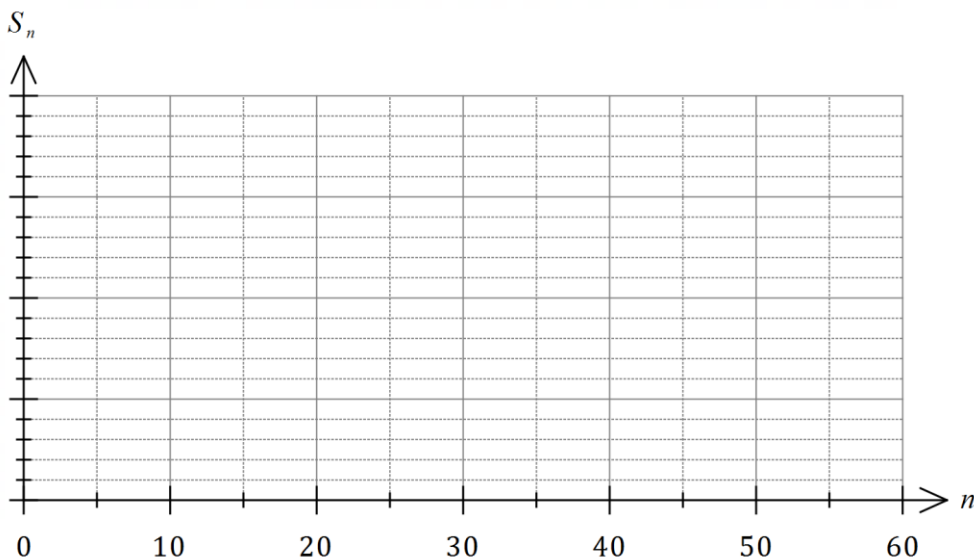
(8 marks)

Saltwater flows steadily into a tank, where it is mixed with existing water. An overflow spout on the tank allows excess water to flow out. The salt concentration in the tank can be modelled by $S_{n+1} = 0.96S_n + 850$, $S_0 = 500$, where S_n is the concentration, in parts per million, after saltwater has been flowing into the tank for n minutes.

(a) Determine S_5 . (1 mark)

(b) Determine the value of n for S_n to first exceed 10 000 ppm. (1 mark)

(c) Plot points to show the salt concentration at 10 minute intervals on the axes below, after first adding an appropriate scale to the vertical axis. (3 marks)



(d) Describe the feature of the plotted points that indicates the salt concentration will eventually reach a steady-state. (1 mark)

(e) Determine, with justification, the steady-state salt concentration. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 17

(7 marks)

Loans are offered by lender *A* at a rate of 10.32% per annum compounded quarterly and from lender *B* at a rate of 10.22% per annum compounded daily over the same duration.

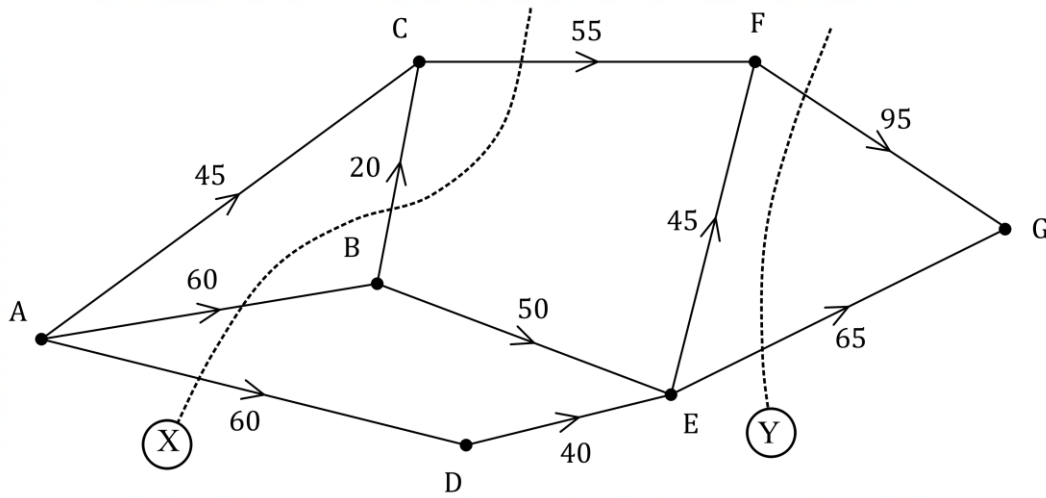
- (a) Calculate the total amount that must be repaid if \$1500 is borrowed from lender *A* for nine months. (2 marks)
- (b) Calculate the total interest that will be charged on a loan of \$90 000 from lender *B* for 30 days. (2 marks)
- (c) Minimising loan interest is the primary goal for a borrower. Calculate the effective interest rate for each lender and hence recommend which should be chosen. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 18

(7 marks)

When a city bypass is closed, traffic that would normally use it is forced to flow through main roads in the city. The edge weights on the directed graph below show the maximum number of vehicles per minute that can travel between junctions (represented by vertices) without causing congestion in the city.



(a) Determine the value of cut X and the value of cut Y . (2 marks)

(b) By listing the capacity of each path, determine the maximum flow of vehicles per hour from A to G . (3 marks)

(c) City engineers recommend taking steps to improve traffic flow between junctions D and E . Determine, with reasoning, the maximum increase in the hourly flow of vehicles from A to G that their plan could achieve. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 19

(8 marks)

A fund with a balance of \$425 000 is used to create an annuity, from which regular withdrawals of \$9500 are to be made at the end of each quarter. Interest at a rate of 6.8% per annum is added to the fund quarterly, just before each withdrawal.

- (a) Use one or more calculations to show that the balance of the fund after one withdrawal is \$422 725. (2 marks)
- (b) Write a recurrence relation to calculate the balance T_n after the n^{th} withdrawal. (2 marks)
- (c) Determine the total interest earned by the fund during its first year of operation. (2 marks)
- (d) The amount of the regular withdrawal from the fund can be modified so that from the outset, the fund is a perpetuity rather than an annuity. Determine the withdrawal required for this to occur. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 20

(8 marks)

For each quarter in the 12 years from 2006 to 2017, a city library calculated its mean number of users per day and used seasonal indices to deseasonalise the data. A snapshot of the data for the year 2015 is shown in the table below.

Quarter	1	2	3	4
Mean daily users	415.4	465.4	462.6	407.0
Deseasonalised mean daily users	436.5	442.7	431.7	439.8

The trend line for the deseasonalised mean daily users is $\hat{n} = 722 - 8.1t$ where t is the quarter and $t = 1$ corresponds to the first quarter of 2006.

- (a) State, with justification, whether the mean number of users per day was increasing or decreasing over the 12 years. (1 mark)

- (b) Calculate the seasonal indices and enter them in the following table. (2 marks)

Quarter	1	2	3	4
Seasonal Index				

- (c) State, with justification, in which quarter the library was usually busiest during the period that data was collected. (2 marks)

- (d) Determine an estimate, to the nearest whole number, for the mean daily users the library experienced in the first quarter of 2017. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 21

(7 marks)

An electronics store advertises a TV for sale at a price of \$5950. Rather than pay this amount in full, the store offers customers a no deposit reducible balance loan with 24 monthly payments of \$281.20. The first repayment is due one month after the customer makes the purchase, just after interest for the month is added to the loan balance.

(a) Determine the total interest paid by customers who buy the TV using the loan. (2 marks)

(b) Determine the annual percentage interest rate that applies to the loan. (2 marks)

A customer decides to buy the TV using a similar reducible balance loan to that offered by the store but financed by their bank at an annual interest rate of 13.98% and over 36 months.

(c) Determine their monthly repayment. (2 marks)

(d) Determine the total interest that the bank will charge on the loan over the 36 months. (1 mark)

End of questions

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF